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Monkeypox: A Mathematical Approach

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Abstract: Monkeypox is a zoonotic disease caused by an orthopoxvirus. It is a smallpox-like disease in humans. It was diagonos in humans in 1970 in the Democratic Republic of the Congo (DRC), it has spread to other regions of Africa (primarily West and Central), and cases outside Africa have emerged in recent years. Since 2003, import- and travel-related spread outside of Africa has occasionally resulted in outbreaks. Interactions/activities with infected animals or individuals are risk behaviors associated with acquiring monkeypox. In this research article, a mathematical model of monkeypox is proposed and analyzed. Here a four diamentional mathematical model is considered. The dynamical behavior of the system is studied analytically. Existence condition and stability analysis are performed. Our aim is to control the disease monkeypox by using control therapeutic approach.

Keywords*:* Monkeypox; Zoonotic disease; Smallpox; Lymphadenopathy.

I. INTRODUCTION

Monkeypox can spread from person to person through contact with an infected person's injuries or scabs that may be found on the skin or mucosal surfaces (such as eyes, mouth, throat, genitalia, anus, or rectum).The virus may spread through respiratory particles, such as from talking, breathing, coughing or sneezing, during close contact.

Monkeypox is a disease caused by a virus in the orthopoxvirus group, which includes smallpox. The monkeypox virusa DNA virus was first identified in monkeys in a lab in Denmark in 1958, but it is more typically found in rodents and other animals. It has caused periodic outbreaks since the 1970s in Central and West Africa, where the virus has been found in several animal species [1].The major hosts of Poxviruses are rodents, rabbits, and non-human primates, which can occasionally be transmitted to humans facilitating the occurrence of human-to-human transmission [2].

Monkeypox is transmitted through prolonged direct contact with someone who has been infected with the virus or through prolonged contact with contaminated objects.

The virus can enter the body through skin lesions, the respiratory tract, eyes, nose or mouth as well as through close contact with body fluids. Monkeypox can also be transmitted from one person to another through droplets from coughing or sneezing, but only with prolonged close contact. Moreover, research has shown that transmission can also occur via the placenta from mother to foetus. Monkeypox has not yet been declared a sexually transmitted infection, although the prolonged direct exposure which is specific to intimate contact and exposure to infected body fluids facilitate the transmission of the disease from one person to another. Currently, the most-at-risk groups are healthcare professionals treating infected people as well as the family members or sexual partners of the infected person [3].

Animals may spread the infection to other animals or to humans, and human infections may spill back into animals. With seasonal increases in animal populations, the risk to humans may correspondingly increase.

The current global outbreak of monkeypox virus infection in humans suggests changes in biologic aspects of the virus, changes in human behavior, or both; such changes might be driven by waning smallpox immunity, relaxation of coronavirus disease 2019 (Covid-19) prevention measures, resumption of international travel, and sexual interactions associated with large gatherings [4].

The symptoms of monkeypox virus are milder than, but similar to those of smallpox. One key difference is that monkeypox causes swollen lymph nodes (lymphadenopathy) while smallpox does not. The swelling can occur in many different locations on the body, or be localized, including lymph nodes of the neck and armpit. The incubation period (time from infection to onset of symptoms) for monkeypox is usually 7−14 days but can range from 5−21 days. In addition to swollen lymph nodes, early signs and symptoms may include fever, headache, muscle aches, exhaustion, backache, and chills. A rash develops on the body within 1 to 3 days (sometimes longer) after the appearance of fever, usually first on the face, then spreading to other parts of the body. The lesions progresses through the stages of macules, papules, vesicles, pustules, and finally scabs before falling off.

We develop a deterministic mathematical model for the monkey pox virus. It is shown that the model undergoes, where the locally stable disease-free equilibrium co-exists with an endemic equilibrium. Furthermore, we determine conditions under which the

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disease-free equilibrium of the model is globally asymptotically stable. Finally, numerical simulations to demonstrate our findings and brief discussions are provided. The findings indicate that isolation of infected individuals in the human population helps to reduce disease transmission [5].

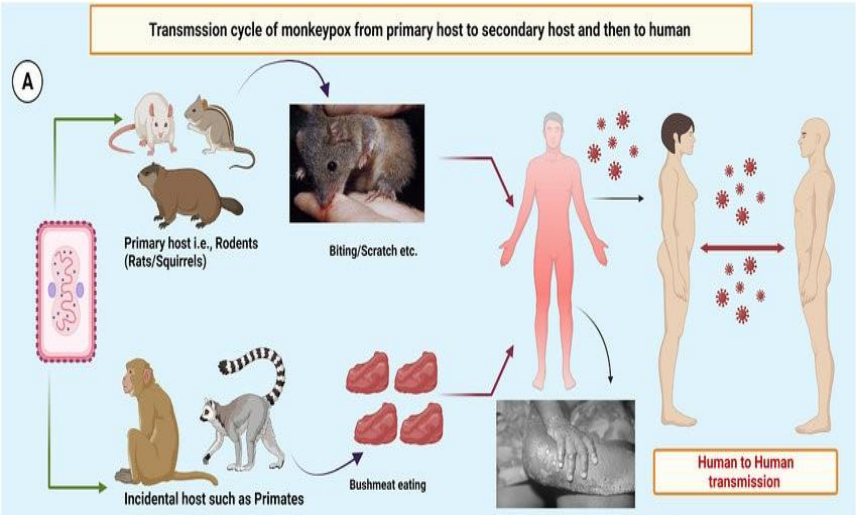

Fig. 1 The figure represents the mechanisms of monkeypox virus transmission [6].

Populations at high risk for MPXV infection are often vaccinated prior to exposure. This has historically included lab workers and clinicians. Routine vaccination is not currently available in endemic countries. Post-exposure vaccination can reduce the risk of infection when given within 4 days of exposure and can reduce the severity of symptoms when given between 4 and 14 days after exposure [7]. However, the time between the onset of fever and the onset of rash has been shown to be longer, and the disease can present as mild or asymptomatic in vaccinated individuals, potentially altering transmission dynamics [8]. The extent of protection against MPXV breakthrough offered by these vaccines remains unclear [9].

There is currently no specific treatment approved for MPXV infection, though there are several antivirals developed to treat smallpox that are being tested, including tecovirimat, brincidofovir, and cidofovir. In a retrospective study of MPX cases in the United Kingdomfrom 2018 to 2021, one of seven patients was treated with tecovirimat and experienced a shorter duration of viral shedding, indicating that antivirals may help reduce the risk of MPXV transmission.

The results of running this model showed that monkeypox can easily spread by human-human transmission if public health controls are neglected, up to 100 new infections could happen per day, with 2% being within the HRG. With the isolation of two out of three cases, transmission can be brought down to a very low level. Assuming a reproductive number (R0) of <1, the mean number of new infections daily in the HRG would be below 1.

II. DEVELOPMENT OF MATHEMATICAL MODEL

We consider that prey population is facing an infectious disease, where the predator feeds on both healthy and infected preys. Let, S_H is the susceptible human population, I_H is the infected human population, S_R is the susceptible reservoir population, I_R is the infected reservoir population.

$$\frac{dS_H}{dt} = \lambda_H - \frac{\alpha_2 I_R S_H}{N_H} - \mu_h S_H$$

$$\frac{dI_H}{dt} = \frac{\alpha_1 I_H S_H}{N_H} - \delta I_H - \mu_H I_H$$

$$\frac{dS_R}{dt} = \lambda_R - \mu_R S_R$$

$$\frac{dI_R}{dt} = \frac{\beta I_R S_R}{N_R} - (\gamma + \mu_R) I_R$$

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where, λ_H is growth rate of susceptible human population, α_1 is infection rate of human to human, α_2 is infection rate of reservoir to human, μ_H is natural death rate of human, δ is death rate due to disease, λ_R is growth rate of susceptible reservoir, $\beta$is infection rate of reservoir, $\mu_R$is natural death rate of reservoir.

III. EXISTENCE AND LOCAL STABILITY ANALYSIS OF THE EQUILIBRIUM POINTS

There are four equilibrium points of the aggregated system, trivial equilibrium point $E_0(0,0,0,0)$,planer equilibrium points $E_1\left(\frac{\lambda_H}{\mu_H},0,\frac{\lambda_R}{\mu_R},0\right), E_2\left(\frac{\lambda_H}{(\alpha_1-\delta)},\frac{\lambda_H(\alpha_1-\delta-\mu_H)}{(\alpha_1-\delta)(\delta+\mu_H)},\frac{\lambda_R}{\mu_R},0\right), E_3\left(\frac{\lambda_H-\alpha_2 I_R}{\mu_H},0,\frac{\lambda_R}{(\beta-\gamma)},\frac{\lambda_R-\mu_R S_R}{(\gamma+\mu_R)}\right)$ and the interior equilibrium point $E^*(S_H^*, I_H^*, S_R^*, I_R^*)$where $S_H^*, I_H^*, S_R^*, I_R^*$ satisfy the following equations

$$S_H^* = \frac{\lambda_H - I_H^*(\delta+\mu_H)}{\mu_H}$$
$$S_R^* = \frac{\lambda_R - (\gamma+\mu_R I_R^*)}{\mu_R}$$

The variational Matrix is given by:

$$V = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$a_{11} = -\frac{I_H}{N_H^2}(\alpha_1 I_H + \alpha_2 I_R) - \mu_H,$
$a_{12} = -\frac{\alpha_1 S_H^2}{N_H^2} + \frac{\alpha_2 S_H I_R}{N_H^2},$
$a_{13} = 0,$
$a_{14} = \frac{\alpha_2 S_H}{N_H}$
$a_{21} = \frac{I_H}{N_H^2}(\alpha_1 I_H + \alpha_2 I_R),$
$a_{22} = \frac{\alpha_1 S_H^2}{N_H^2} - \frac{\alpha_2 S_H I_R}{N_H^2} - (\delta - \mu_H),$
$a_{23} = 0,$
$a_{24} = \frac{\alpha_2 S_H}{N_H}$
$a_{31} = 0,$
$a_{32} = 0,$
$a_{33} = -\frac{\beta I_R^2}{N_R^2} - \mu_R,$
$a_{34} = -\frac{\beta S_R^2}{N_R^2}$
$a_{41} = 0,$
$a_{42} = 0,$
$a_{43} = \frac{\beta I_R^2}{N_R^2},$
$a_{44} = -\frac{\beta S_R^2}{N_R^2} - (\gamma - \mu_R)$

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Lemma 1: The equilibrium point $E_0(0,0,0,0)$ is always unstable.

Lemma 2: The system around the axial equilibrium point $E_1(\frac{\lambda_H}{\mu_H}, 0, \frac{\lambda_R}{\mu_R}, 0)$ is conditionally stable.

Proof: The eigen values about$E_1$are $\lambda_1^4 = -\mu_H, \lambda_2^4 = \alpha_1 - \delta - \mu_H$, $\lambda_3^4 = -\mu_R$, $\lambda_4^4 = \beta - \gamma - \mu_R$. As $\lambda_1^4$and λ_3^4 is always less than zero, the equilibrium point $E_1$is stable when $\lambda_2^4 = \alpha_1 - \delta - \mu_H < 0$ and $\lambda_4^4 = \beta - \gamma - \mu_R < 0$. So, the equilibrium point $E_1$is stable when $\alpha_1 > \delta + \mu_H$ and $\beta < \gamma + \mu_R$.

Lemma 3: The system around the planer equilibrium point $E_2(\frac{\lambda_H}{(\alpha_1-\delta)}, \frac{\lambda_H(\alpha_1-\delta-\mu_H)}{(\alpha_1-\delta)(\delta+\mu_H)}, \frac{\lambda_R}{\mu_R}, 0)$ is conditionally stable.

Proof: The eigen values about$E_2$are given by the equations $\lambda^2 - \lambda(a_{11} + a_{22}) + (a_{11}a_{22} - a_{12}a_{21}) = 0 \; and \; \lambda^2 - \lambda(a_{33} + a_{44}) + (a_{33}a_{44} - a_{34}a_{43}) = 0$

The two eigen values of $\lambda^2 - \lambda(a_{11} + a_{22}) + (a_{11}a_{22} - a_{12}a_{21}) = 0$ will have the –ve real part if $\alpha_1 > \delta + \mu_H$

The two eigen values of $\lambda^2 - \lambda(a_{33} + a_{44}) + (a_{33}a_{44} - a_{34}a_{43}) = 0$ will have the –ve real part if $\beta < \gamma + \mu_R$

Lemma 4: The system around the planer equilibrium point $E_3(\frac{\lambda_H - \alpha_2 I_R}{\mu_H}, 0, \frac{\lambda_R}{(\beta-\gamma)}, \frac{\lambda_R - \mu_R S_R}{(\gamma+\mu_R)})$ is conditionally stable.

Proof: The eigen values about$E_3$are given by the equations $\lambda^2 - \lambda(a_{11} + a_{22}) + (a_{11}a_{22} - a_{12}a_{21}) = 0 \; and \; \lambda^2 - \lambda(a_{33} + a_{44}) + (a_{33}a_{44} - a_{34}a_{43}) = 0$

The two eigen values of $\lambda^2 - \lambda(a_{11} + a_{22}) + (a_{11}a_{22} - a_{12}a_{21}) = 0$ will have the –ve real part if $\alpha_1 - \frac{\alpha_2 I_R}{S_H} - \delta - \mu_H < 0$

The two eigen values of $\lambda^2 - \lambda(a_{33} + a_{44}) + (a_{33}a_{44} - a_{34}a_{43}) = 0$ will have the –ve real part if $\frac{\beta I_R^2}{N_R^2} + \gamma + \mu_R - \frac{\beta S_R^2}{N_R^2} > 0$

Lemma 5: The system around the interior equilibrium point $E^*(S_H^*, I_H^*, S_R^*, I_R^*)$is conditionally stable.

Proof: The equilibrium is stable when

(i) $I_H^*(\alpha_1 I_H^* + \alpha_2 I_R^*) + \mu_H N_H^{*^2} > Max[\{\alpha_1 S_H^{*^2} - \alpha_2 I_R^* S_H^* - N_H^{*^2}(\delta + \mu_H)\}, \{\frac{\mu_H(\alpha_1 S_H^{*^2} - \alpha_2 I_R^* S_H^*)}{\delta + \mu_H}\}]$

(ii) $\beta I_R^{*^2} + (\gamma + 2\mu_R) N_R^{*^2} - \beta S_R^{*^2} > 0$

(iii) $(\beta I_R^{*^2} + \mu_R N_R^{*^2})(\gamma + \mu_R) - \mu_R \beta S_R^{*^2} > 0$

(iv) $(\beta I_R^{*^2} + \mu_R N_R^{*^2}) > Max[\{\beta S_R^{*^2} - (\gamma + \mu_R) N_R^{*^2}\}, \{\frac{\mu_R \beta S_R^{*^2}}{\gamma + \mu_R}\}]$

IV. CONCLUSION

The last two years have been really challenging the COVID-19 pandemic changed so much about the way we lived and worked. Now, just as we're starting to ease back into regular life, we're hearing media stories about monkeypox as an emerging threat. But monkey pox is a rare disease. It's spread through close contact, like kissing and sex, though we're still learning about how monkey pox spreads in humans. The best way to protect yourself is to avoid contact with people who are infected.

A deterministic mathematical model was developed for the transmission dynamics of Monkeypox virus. It was shown that the model is mathematically and epidemiologically well posed and that the solution of the model at all time will be positive. The equilibrium of the model equation was obtained and analysed. The system was shown to have one unique endemic equilibrium which is stable when $R_0 < 1$. Numerical simulation was carried it to emphasize the role of weak, medium and strong immune system on some epidemiological states, as well as the effect of contagion and vaccination rates on the prevalence and susceptible respectively.

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